ABSTRACT

Incontro Nazionale di Analisi Ipercomplessa

Progetto INdAM
Teoria delle funzioni ipercomplesse e applicazioni

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Giulio Binosi (Università di Trento)
Partial slice regularity, Almansi decomposition and Fueter's theorem in $\mathbb{H}^n$

Given a circular set $\Omega_D \subset \mathbb{H}^n$, let $\mathcal{S}_h$ (resp. $\mathcal{SR}_h$) be the set of function which are slice (resp. slice regular) w.r.t. x_h. We characterize the set $\mathcal{S}(\Omega_D) \cap \mathcal{S}_h(\Omega_D)$ in terms of the inducing stem functions and the set $\mathcal{SR}_h(\Omega_D)$ as x_h-slice functions, solutions of the operator $\frac{\partial}{\partial x_h^c}$. We generalize the spherical derivative of a function $f \in \mathcal{S}(\Omega_D)$, through which we define the components $\{\mathcal{S}_K^m(f)\}_{K \in \mathcal{P}(m)}$, which possess various properties, including harmonicity. They will provide an Almansi-type decomposition of the form

$$f = \sum_{K \in \mathcal{P}(m)} (-1)^{|K^c|} (\overline{x})_{K^c} \mathcal{S}_K^m(f).$$

As consequences of the decompositions we find, among others, the biharmonicity of every slice regular function and a generalization of Fueter's theorem in $\mathbb{H}^n$.

Cinzia Bisi (Università di Ferrara)
Invariants and automorphisms of slice regular functions

Let A be one of the following algebras: $\mathbb{C}, \mathbb{H} = \mathbb{R}_2, \mathbb{R}_3, \mathbb{O}$. For the algebra A, the automorphism group $\mathrm{Aut}(A)$ and its invariants are well known.

The talk will describe the invariants of the automorphism group of the algebra of slice regular functions over $A = \mathbb{H} = \mathbb{R}_2$ and over $A = \mathbb{R}_3$. Time permit-

ting, the talk will also cover the case $A = \mathbb{O}$, which requires more sophisticated techniques.

Giulia Dileo (Università di Bari)
On a special class of manifolds fibering over quaternionic Kähler manifolds

I will introduce 3-(α, δ)-Sasaki manifolds, a special class of manifolds generalizing 3-Sasaki manifolds, and locally fibering over a quaternionic Kähler manifold of vanishing, positive or negative scalar curvature, according to $\delta = 0$, $\alpha\delta > 0$, or $\alpha\delta < 0$. A central role in the geometry of these manifolds is played by a canonical metric connection with skew torsion. Both the Riemannian curvature and the curvature of the canonical connection can be investigated, with special attention to conditions of strongly positive curvature, and the behavior in the homogeneous case. This is a joint work with Ilka Agricola and Leander Stecker.

Riccardo Ghiloni (Università di Trento)
On the zeros of polynomials in several quaternionic variables

We present some basic properties of the zero sets of slice regular polynomials in several quaternionic variables.

Anna Gori (Università di Milano)
Moduli spaces of conformal classes for flat tori of higher dimension and their conformal multiplication

Motivated by the theory of complex multiplication of abelian varieties, in this paper we study the conformality classes of flat tori in $\mathbb{R}$ and investigate criteria to determine whether a n-dimensional flat torus has non trivial (i.e., bigger than $\mathbb{Z}^* = \mathbb{Z} \setminus \{0\}$) semigroup of conformal endomorphisms (the analogs of isogenies for abelian varieties). We then exhibit several geometric constructions of tori with this property and study the class of conformally equivalent lattices in order to describe the moduli space of the corresponding tori.
This is a joint work with Alberto Verjovsky and Fabio Vlacci.

Marco Maggesi (Università di Firenze)
Explaining basic quaternionic analysis to the computer

Computers can manipulate symbolic mathematical expressions and logical formulas. Using this ability, advanced mathematical results can be formally checked by combining automatic and interactive machine theorem proving. We will discuss some specific aspects of the computer formalization of Quaternionic Analysis and present some of our experiments and results in this context

Alessandro Perotti (Università di Trento)
Wirtinger operators for functions of several quaternionic variables

The aim of the talk is to introduce Wirtinger operators for functions of several quaternionic variables. These operators are linear differential operators with properties analogous to the ones satisfied by the Wirtinger derivatives of several complex variables. Due to the non-commutativity of the variables, Wirtinger operators turn out to be of higher order. In spite of that, they commute each other and satisfy a Leibniz rule for products. Moreover, they characterize the class of slice-regular quaternionic functions.

Giulia Sarfatti (Università Politecnica delle Marche)
Zero sets and Nullstellensatz type theorems for slice regular polynomials

The Hilbert's Nullstellensatz can be regarded as a generalization of the Fundamental Theorem of Algebra to the case of polynomials in several variables and is a central result in Algebraic Geometry. In this seminar, we will discuss its possible extensions to the setting of slice regular polynomials in several quaternionic variables. The results presented, obtained in collaboration with Anna Gori and Fabio Vlacci, begin with the study of the relation between zeros and factorization properties of slice regular polynomials.

For polynomials in two variables, thanks to symmetry properties of their vanishing sets, we get a more geometric version of the so called Strong Nullstellensatz. Moreover, using a suitable Dieudonné determinant, we define a non-commutative resultant, providing a tool to investigate the existence of common zeros of two polynomials.